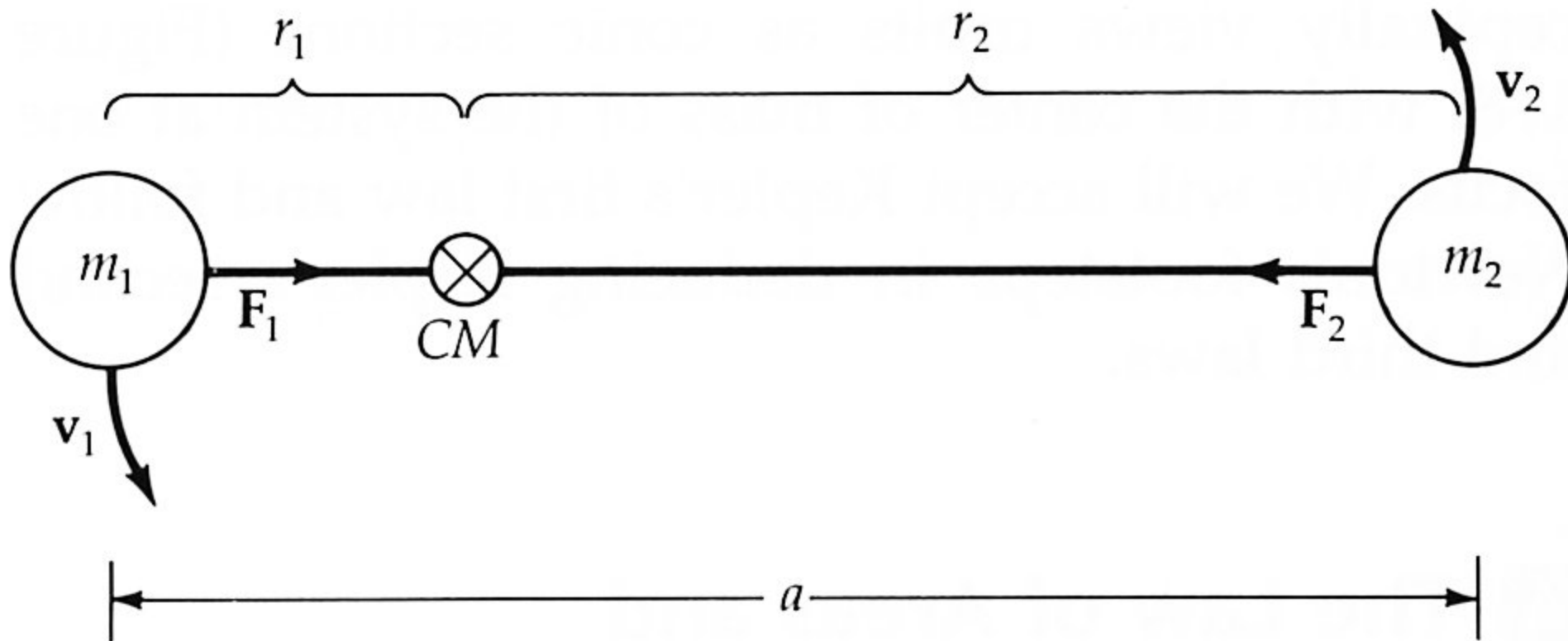


# Stellar Masses

- Binary systems
- Kepler's 3<sup>rd</sup> Law
- Visual binaries
- Spectroscopic binaries

# Binary Systems

- consider two stars with masses  $M_1$  and  $M_2$  in circular orbits around their centre of mass (CM)
- radius of each orbit is  $r_1$  and  $r_2$  respectively and the total separation is  $a$
- can use Newton's Laws and circular motion to determine masses



Zeilik Fig 1-14

# Circular Motion

$$F_1 = \frac{M_1 v_1^2}{r_1} = \frac{4\pi^2 M_1 r_1}{P^2}$$

and

$$F_2 = \frac{M_2 v_2^2}{r_2} = \frac{4\pi^2 M_2 r_2}{P^2}$$

where  $P$  is the period which is the same for both stars

# Centre of Mass

- definition of centre of mass means

$$M_1 r_1 = M_2 r_2$$

# Newton's Law of Gravity

$$F_1 = F_2 = \frac{GM_1M_2}{a^2}$$

where

$$a = r_1 + r_2$$

# Newton's form of Kepler's Third Law

- combining these three equations gives

$$\frac{4\pi^2 M_1 r_1}{P^2} = \frac{GM_1 M_2}{a^2}$$

$$P^2 = \frac{4\pi^2 a^2 r_1}{GM_2}$$

Eliminate  $r_1$  using

$$a = r_1 + r_2 = r_1 + \frac{M_1}{M_2} r_1 = \left( \frac{M_1 + M_2}{M_2} \right) r_1$$

so

$$P^2 = \frac{4\pi^2 a^3}{G(M_1 + M_2)}$$

and

$$M_1 + M_2 = \frac{4\pi^2 a^3}{GP^2}$$

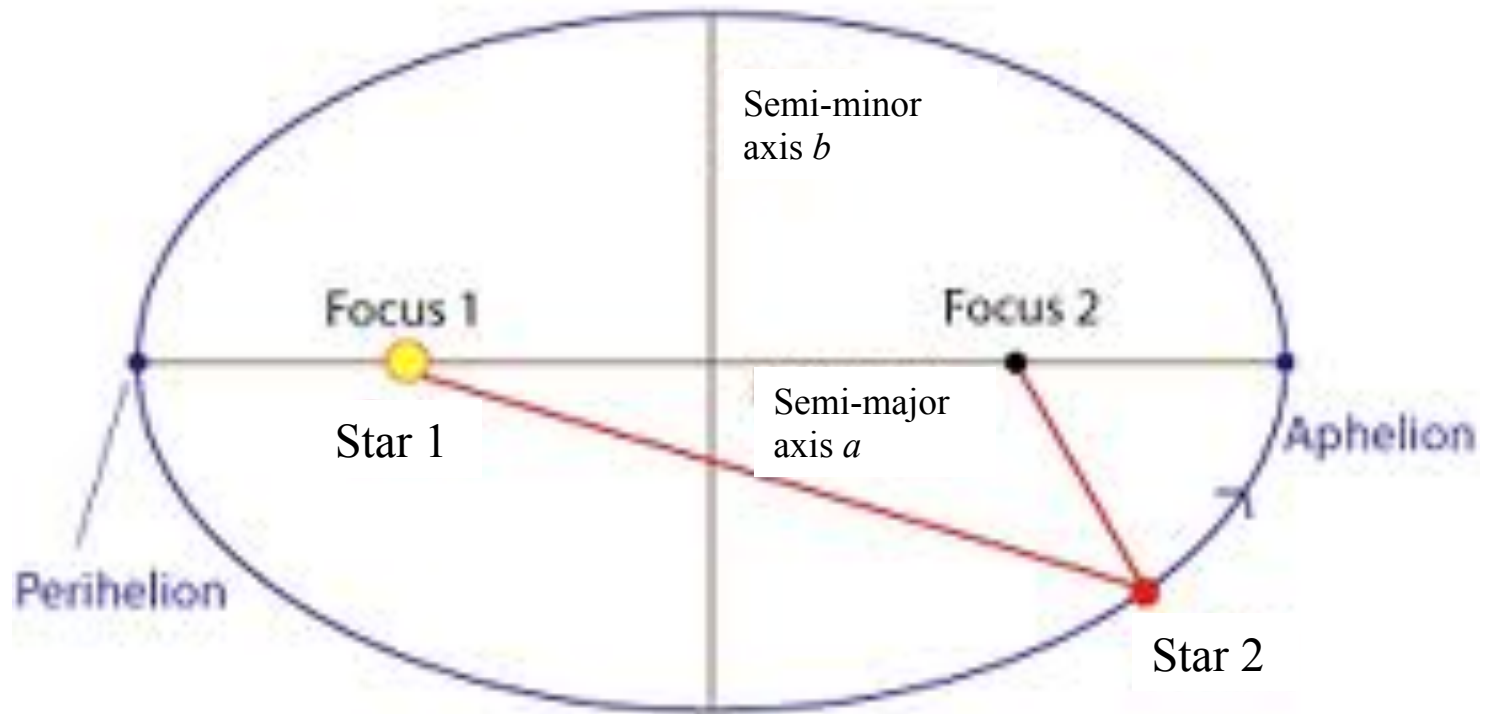
# Real Orbits

- orbits are generally elliptical and described by their semi-major axis  $a$  and semi-minor axis  $b$
- eccentricity is defined by

$$e = \frac{\sqrt{a^2 - b^2}}{a}$$

i.e.  $e = 0 \Rightarrow$  circular orbit

- Newton's form of Kepler's third law also applies to elliptical orbits

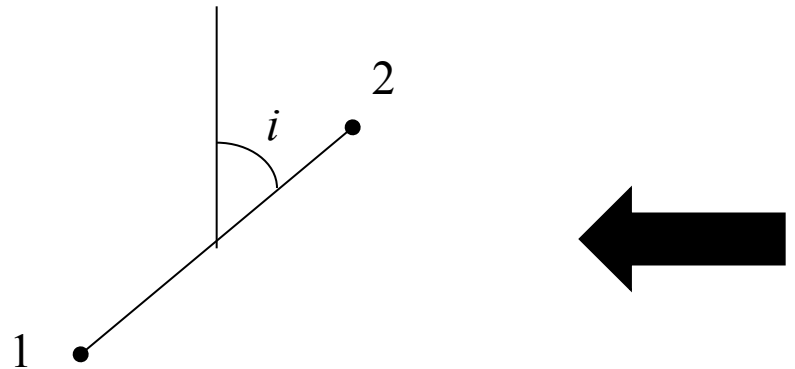


# Orbital Inclination

- in general the orbital plane of a binary system will be inclined by some angle  $i$  to the plane of the sky:

$i = 0^\circ \Rightarrow$  face on

$i = 90^\circ \Rightarrow$  edge on

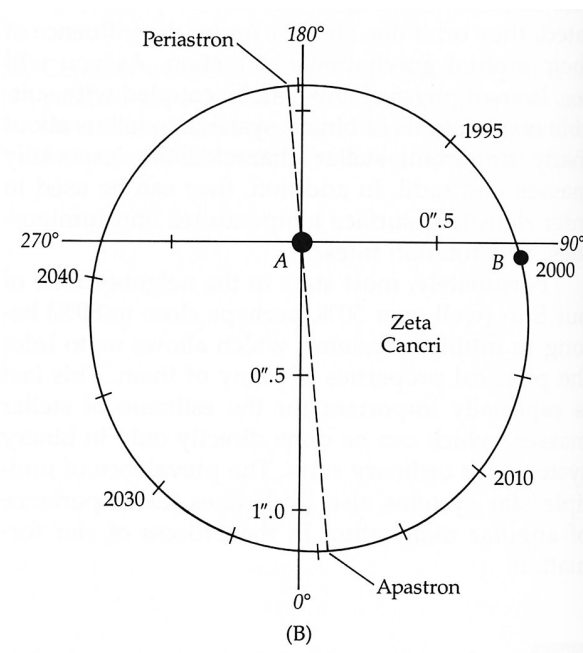
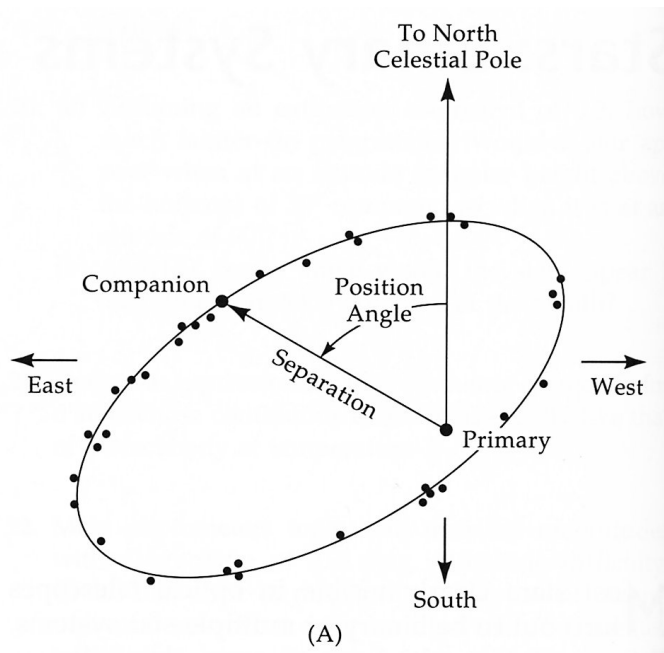


# Types of Binary System

- Visual binaries
  - Two stars spatially resolved on the sky in orbit around each other
- Spectroscopic binaries
  - Two stars not spatially resolved, but orbital motion revealed through periodic Doppler shifts of their spectral lines

# Visual Binaries

- Can measure sum of masses from Kepler's law and ratio of masses from ratio of semi-major axes and hence can solve for individual masses



# Masses from Spectroscopic Binaries

- for circular orbits the orbital velocities are

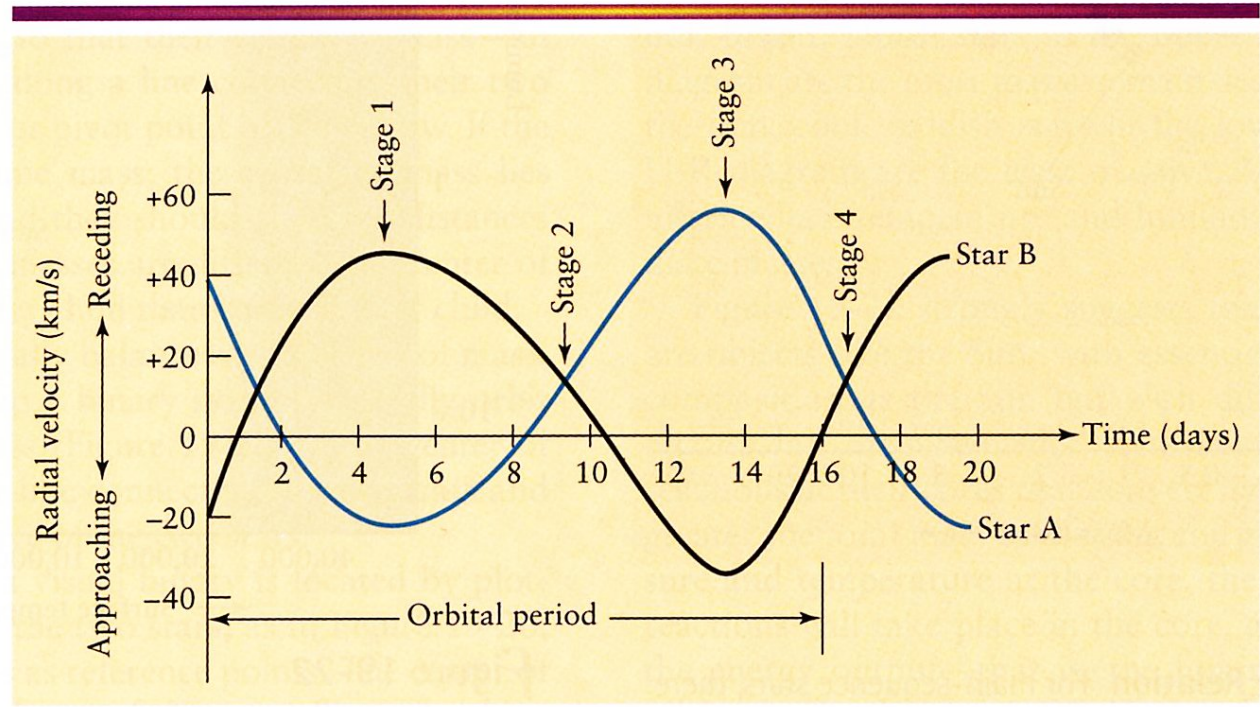
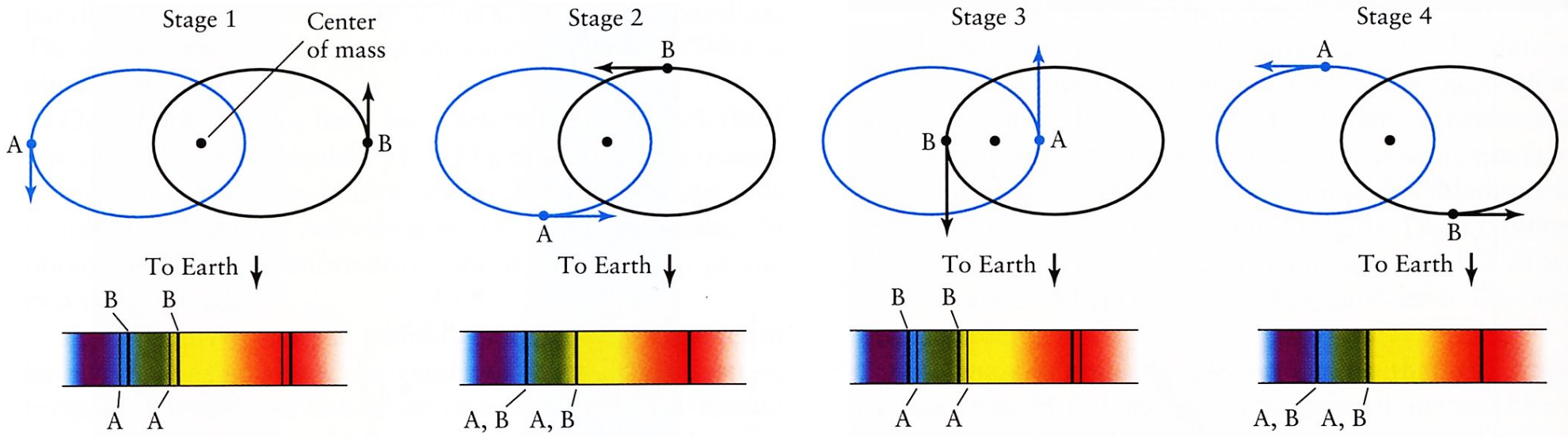
$$v_1 = \frac{2\pi r_1}{P} \quad \text{and} \quad v_2 = \frac{2\pi r_2}{P}$$

- for inclination angle  $i$  the observed radial velocities are

$$v_{r1} = v_1 \sin i \quad \text{and} \quad v_{r2} = v_2 \sin i$$

- If we see lines from both stars can determine mass ratio from

$$\frac{v_{r1}}{v_{r2}} = \frac{v_1}{v_2} = \frac{r_1}{r_2} = \frac{M_2}{M_1}$$



Also

$$a = r_1 + r_2 = \frac{P}{2\pi} (v_1 + v_2) = \frac{P}{2\pi} \left( \frac{v_{r1} + v_{r2}}{\sin i} \right)$$

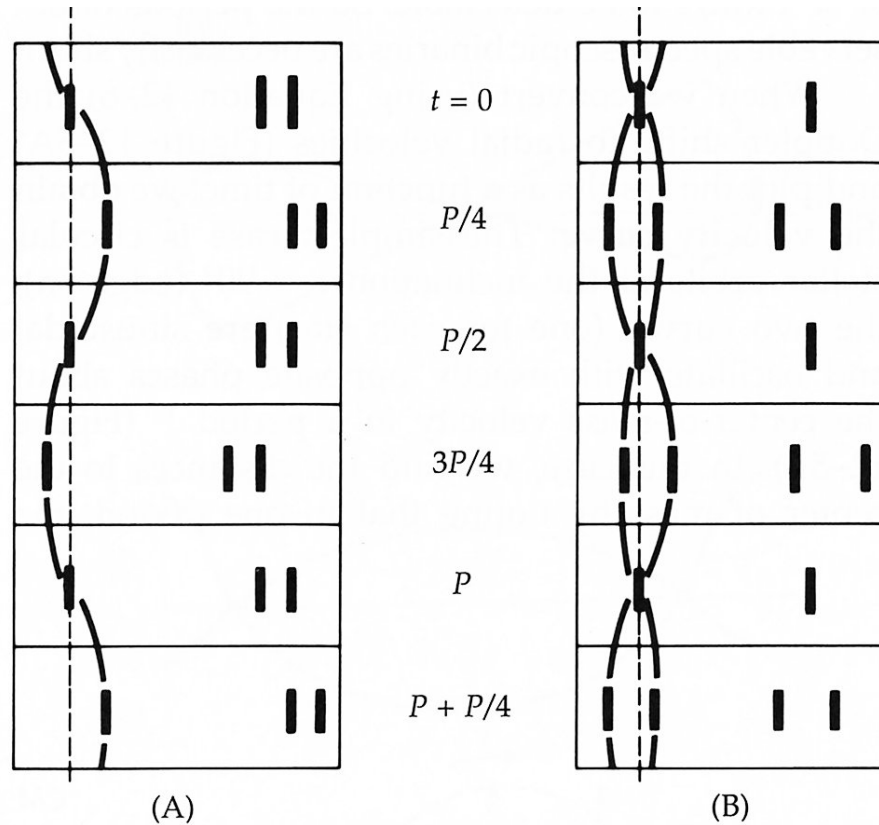
so from Kepler's law

$$M_1 + M_2 = \frac{4\pi^2 a^3}{GP^2} = \frac{P}{2\pi G} \left( \frac{v_{r1} + v_{r2}}{\sin i} \right)^3$$

i.e. only a lower limit to the sum of the masses

# Single-lined Spectroscopic Binaries

- only one spectrum is observed say  $v_{r1}$



Zeilik Fig 12-4

- so eliminate  $v_{r2}$

$$M_1 + M_2 = \frac{P}{2\pi G} \left( \frac{v_{r1} + \frac{M_1}{M_2} v_{r1}}{\sin i} \right)^3$$

$$M_1 + M_2 = \frac{Pv_{r1}^3}{2\pi G} \left( \frac{M_1 + M_2}{M_2} \right)^3$$

$$\text{so } \frac{M_2^3 \sin^3 i}{(M_1 + M_2)^2} = \frac{Pv_{r1}^3}{2\pi G}$$

i.e. if we can estimate  $M_1$

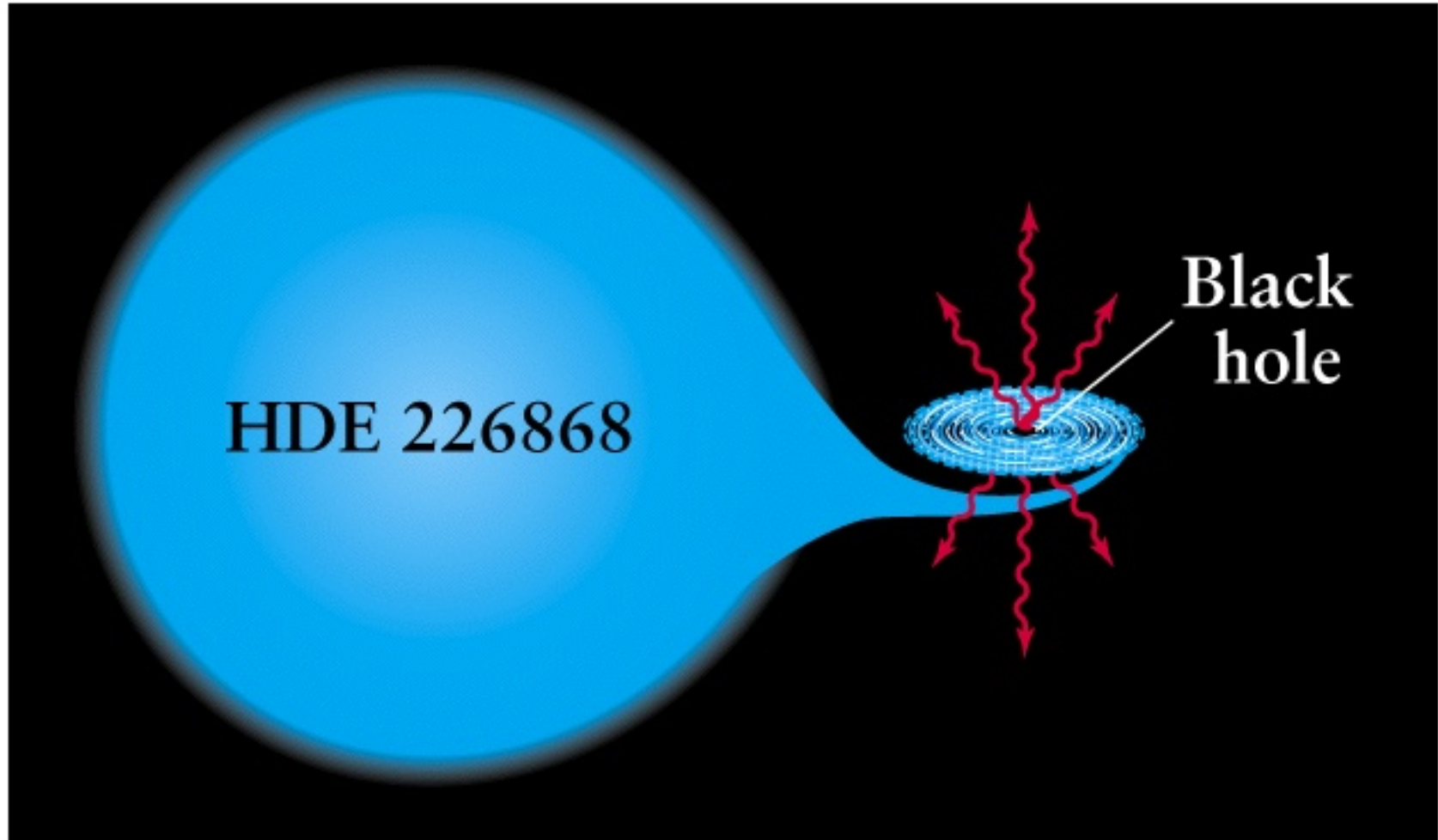
we can constrain  $M_2$

$$M_1 + M_2 = \frac{Pv_{r1}^3}{2\pi G} \left( \frac{M_1 + M_2}{M_2 \sin i} \right)^3$$

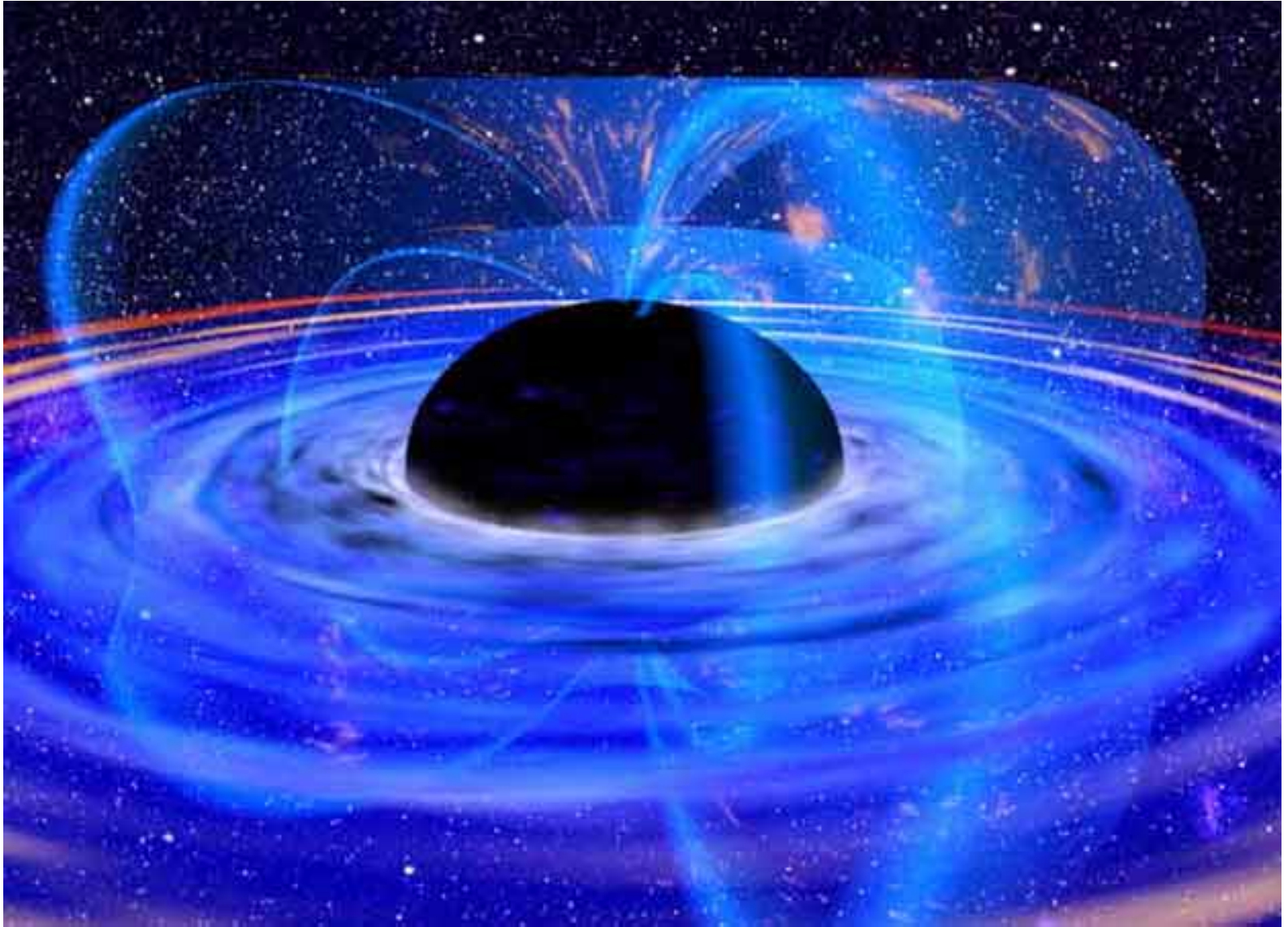
so 
$$\frac{M_2^3 \sin^3 i}{(M_1 + M_2)^2} = \frac{Pv_{r1}^3}{2\pi G}$$

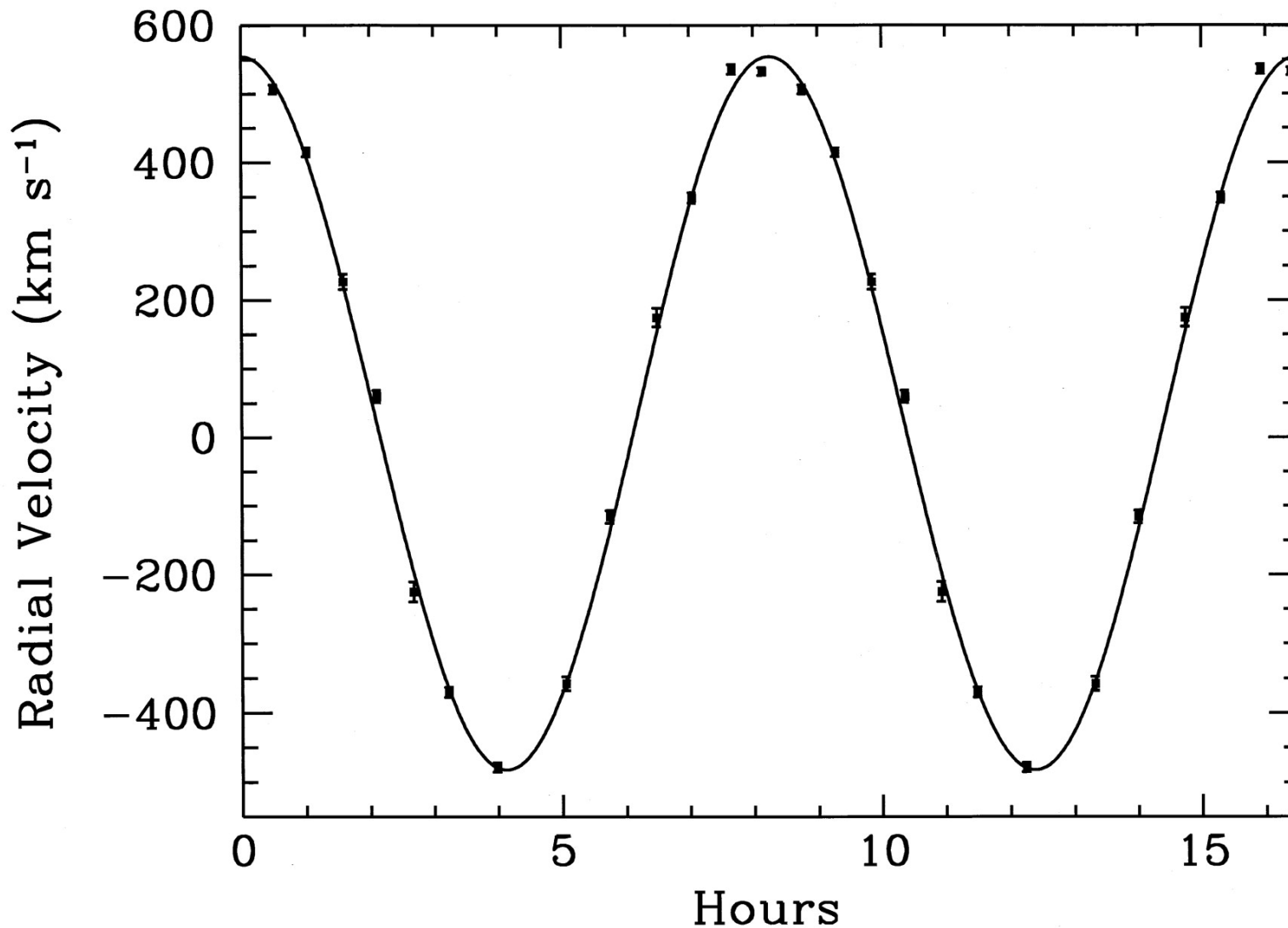
i.e. if we can estimate  $M_1$

we can constrain  $M_2$



**a**





Radial-velocity curve of the visible star in the X-ray binary GS 2000 + 25  
Fillipenko et al. (1999) [www.pnas.org/content/96/18/9993.full](http://www.pnas.org/content/96/18/9993.full)  
Shows that invisible compact companion star is a 5 solar mass black hole

# Summary

- visual binaries provide accurate masses, but not many known
- spectroscopic binaries only usually constrain the masses with inclination the greatest uncertainty unless the system is eclipsing
- spectroscopic binaries used to find black holes and planets orbiting other stars